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A superspace approach to branes and supergravity*

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Abstract: Recent developments in string and M theory rely heavily on supersymmetry suggesting that a revival of superspace techniques in ten and eleven dimensions may be advantageous. Here we discuss three topics of current interest where superspace is already playing an important role and where an improved understanding of superspace might provide additional insight into the issues involved.

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1. Introduction

It is well-known that besides the string (the $p = 1$ brane) itself also branes of other dimensionality ($d = p+1$) play a central role in the non-perturbative structure of the theory. Using Dp-branes and T5-branes (having respectively vector and second rank antisymmetric tensor potentials on the world surface) non-perturbative relations between all string theories can be established as well as a connection with 11d supergravity. This hints at the existence of an underlying more profound formulation of the whole theory. Some aspects of this so called M theory are captured by M(atrix) theory which has its origin in D-particle (D0-brane) physics, but have many features in common with the 11d membrane (see e.g. the talk by B. de Wit).

The three topics discussed below are related to the role superspace is playing in this context. We start by discussing one of the direct implications of formulating the fundamental theory in terms of M(atrix) theory, namely the presence of higher order corrections (e.g. R^n) in 11d supergravity (for a recent review see [1]). Here we will have reason to recall certain facts in 10d supergravity established in the 1980's [2, 3, 4, 5]. Then we review some recent results concerning the realization of κ-symmetry for various branes focusing on the 11d membrane ($p = 2$), the D3-brane in 10d type IIB supergravity, and the T5-brane in 11d superspace. In the latter case we present a new action [6] which avoids some of the problematic features of the lagrangian constructed previously in [7]. As a final topic we consider the generalization of this setup, based on bosonic world sheets embedded into target superspaces, to a situation where both target and the embedded world surface are superspaces [8, 9, 10]. In [10] the relation between non-linearities of the tensor dynamics on the world sheet and non-linearly realized (super)symmetries is analyzed. This paper also contains a detailed account of general embeddings plus some new results for T5-branes in 7d.

2. Superspace and higher order corrections in 11d

One property of M theory that has been highlighted by the recent developments in M(atrix) theory is the higher order corrections in terms of e.g. the curvature tensor that must occur in the low energy supergravity lagrangian in 11d. Since 11d supersymmetry is tremendously restrictive (there is only one multiplet whose spin content does not exceed spin two, a cosmological term is not possible [11] , etc) but also very messy to deal with in terms of component fields, it might be worth the effort to develop further the superspace techniques that were introduced in the 1970's. The heart of the matter is the question of how to realize supersymmetry off-shell. This is fairly well understood in the case of $N = 1$ supersymmetry in 10d [2, 3, 4, 5], a subject that we will therefore have reason to come back to.

The field content of 11d supergravity is ${e_m}^a$, ψ_m^α, c_{mnp} where $m, n, ..$ are 11d vector indices and $\alpha = 1, .., 32$ is a Majorana spinor index. These fields describe 128 bosonic and 128 fermionic degrees of freedom. The field equations are obtained from the corresponding superfields and their super-Bianchi identities. To this end we combine ${e_m}^a$ and ψ_m^α into the supervielbein ${E_M}^A$ where the superworld index $M = (m, \mu)$ and the supertangent space index $A = (a, \alpha)$. Normally one introduces also a superfield C_{MNP} with c_{mnp} as its first component. However, it was recently clarified [12] that this is not necessary, and we will refrain from doing so. Then the field strengths

are just the supercurvature $R_A{}^B$ and the supertorsion $T^A = DE^A = dE^A - E^B\omega_B{}^A$. Their Bianchi identities read $DT^A = E^B R_B{}^A$ and $DR_A{}^B = 0$ but in fact it is only necessary to consider the first identity since the second one is automatically satisfied. The meaning of "solving the Bianchi identities" is as follows. By subjecting the torsion components ($T_{\alpha\beta}{}^c$, etc) to certain constraints the Bianchi identities cease to be identities and become equivalent to the field equations. One should remember however that a given set of gauge fields can often be given a variety of different kinds of dynamics, related either to different sets of constraints or to differences in the Bianchi identities. As shown recently by Howe [12] (on-shell) 11d supergravity is obtained from the *single* constraint (Γ^c is an 11d Dirac matrix)

$$T_{\alpha\beta}{}^c = 2i(\Gamma^c)_{\alpha\beta} \tag{2.1}$$

which is invariant under super-Weyl rescalings. Furthermore, no off-shell formulation of this theory is known.

This situation should be compared to what is known for $N = 1$ supergravity in 10d. When the Bianchi identities are solved on-shell [13] one finds that all physical fields appear at different θ levels in a scalar superfield denoted $\Phi(Z)$, where $Z = (x, \theta)$. Note that 10d supergravity contains a scalar and a spinor, apart from $e_m{}^a,\ \psi_m^\alpha,\ B_{mn}$. In contrast to 11d, one must in this case constrain several torsion components. This theory can be coupled to superYang-Mills using the same torsion constraints [14] and the Bianchi identity $DH = trF^2$ for the three-form $H = dB$. In fact, as proven in [15] even the R^2 term needed for anomaly cancellation can be dealt with without altering the constraints.

However, also R^4 and higher terms are present in the field theory of the low energy 10d superstring. In general such terms cannot be incorporated into the superspace Bianchi identities if the on-shell constraints are used. Fortunately, in this case it is known how to proceed since the off-shell field content is known. It consists of a superconformal gravity multiplet and an unconstrained scalar auxiliary superfield w [2]. To account for this new superfield the constraints must be modified to [3,4]

$$T_{\alpha\beta}{}^c = 2i(\Gamma^c)_{\alpha\beta} + 2i(\Gamma^{c_1\ldots c_5})_{\alpha\beta}X^c{}_{c_1\ldots c_5} \tag{2.2}$$

where X is in the representation 1050^+ of $so(1,9)$ appearing at level θ^4 in w.

All higher order corrections, like R^4, that can occur must be compatible with supersymmetry and fit somewhere in the solution of the Bianchi identities that follow from the off-shell constraints above. E.g. the R^4 term related by supersymmetry (see [16]) to the anomaly term BX_8 can be added as follows [5]:

$$S^{D=10,N=1} = -\frac{1}{(\kappa_{10})^2}\int d^{10}x d^{16}\theta E\Phi(w + c) \tag{2.3}$$

where c is a constant proportional to $\zeta(3)$, and where the w term is the kinetic one and the c term is the supersymmetrization of R^4. E is the superspace measure.

Turning to 11d the situation changes dramatically since it is not known how to solve the Bianchi identities off-shell or how to write down an off-shell action in components. This makes it very hard to address questions in 11d supergravity concerning the higher order corrections. We will here introduce the equivalent of w in 10d into the 11d supertorsion by means of the relaxed constraint

$$T_{\alpha\beta}{}^c = 2i\Gamma^c{}_{\alpha\beta} + 2i\Gamma^{d_1d_2}{}_{\alpha\beta}X^c{}_{d_1d_2} + 2i\Gamma^{d_1\ldots d_5}{}_{\alpha\beta}X^c{}_{d_1\ldots d_5} \tag{2.4}$$

where the tensors in the last two terms are in the representations 429 and 4290 which appear at level θ^4 in an unconstrained 11d scalar superfield. A preliminary analyzes of the Bianchi identities indicates an "off-shell situation" where new terms appear in the torsion which could account for the higher order corrections. In particular terms generated by anomalies and the presence of branes should be investigated. Note that the T5-brane produces in the supersymmetry algebra a five-form central charge, a fact that should be compared to the extra terms in the torsion.

Further studies will hopefully tell if these techniques can be utilized in the endeavour to extract an 11d supergravity theory from M(atrix) theory or perhaps directly from the 11d branes.

3. κ SYMMETRIC BRANES AS BOSONIC SURFACES

The relevant branes in 11d are the membrane and the T5-brane, while in 10d type II theories there are also Dp-branes intermediate between p-branes and T-branes. As we will see below, for branes with vector or tensor fields propagating on the world surface κ-symmetry is technically more complicated than for ordinary p-branes.

Let us as an example of an ordinary p-brane consider the 11d membrane [17]. After the elimination (see [18]) of the independent world sheet metric by means of its algebraic field equation the action reads:

$$S_3 = \int d^3\xi[-\sqrt{-detg} - \varepsilon^{ijk} B_{ijk}] \tag{3.1}$$

where g_{ij} is the pull-back of the target space metric, i.e. $g_{ij} = \Pi_i^a \Pi_j^b \eta_{ab}$, and the ξ^i's are three bosonic coordinates on the world sheet. The background superfields $E_M{}^A$ and B_{MNP} of 11d supergravity enter via the pull-backs $\Pi_i^A = \partial_i Z^M E_M{}^A$ and $B_{ijk} = \Pi_i^A \Pi_j^B \Pi_k^C B_{CBA}$. In order for this action to be supersymmetric the number of bosonic (here 11-3=8) and fermionic ($32 \times \frac{1}{2}$) world sheet on-shell degrees of freedom must match. This requires the presence of the local fermionic κ-symmetry giving another factor of $\frac{1}{2}$ in the fermionic count. Its existence relies on the possibility to construct a projection operator $\frac{1}{2}(1+\Gamma)$ with $\Gamma^2 = 1$. Here $\Gamma = \frac{1}{6\sqrt{-g}}\epsilon^{ijk}\Gamma_{ijk}$ where Γ_{ijk} is the pull-back of $\Gamma_{abc} = \Gamma_{[a}\Gamma_b\Gamma_{c]}$.

This structure can be found also in the case of Dp-branes [19,20], but is now more involved due to the presence of the field strength F_{ij}. E.g. the D3-brane in the 10d type IIB theory has an action that reads

$$S_{D3} = -\int d^4\xi \sqrt{-det(g + e^{-\frac{\phi}{2}}\mathscr{F})} + \int e^{\mathscr{F}} C \tag{3.2}$$

In this case there are, apart from the dilaton ϕ, two kinds of background potentials B and C, coming from the NS-NS and the R-R sector, respectively. In S_{D3}, $\mathscr{F}_{ij} = F_{ij} - B_{ij}$ where B_{ij} is the pullback of B_{MN} in the 10d IIB target space theory. The last term in the action is constructed as a formal sum of forms of different rank and the integral is supposed to pick up only the four-form in this case. That the action is κ-symmetric can then be shown using the Γ matrix ($\Gamma^2 = 1$) [19, 20]

$$\Gamma = \frac{\epsilon^{ijkl}}{\sqrt{-det(g+\mathscr{F})}}(\frac{1}{24}\Gamma_{ijkl}I - \frac{1}{4}\mathscr{F}_{ij}\Gamma_{kl}J + \frac{1}{8}\mathscr{F}_{ij}\mathscr{F}_{kl}) \tag{3.3}$$

The $SL(2;Z)$ symmetry of the IIB theory mixes the B and C potentials and indeed a more symmetric version of the action exists [21]. In that version all background potentials have associated world sheet field strengths to which they couple as $F - B$.

The third case to be discussed here is the T5-brane in 11d. This brane has an additional complication in that the three-form field strength is self-dual. Finding a covariant action for such a field has been a long-standing problem. A first attempt at a solution was given recently by Bandos et al in [7]) and involves an auxiliary scalar field a entering the lagrangian through a factor $\frac{1}{(\partial a)^2}$. Objections against using such an action in quantum calculations have been formulated (see also [22]). Another action that does not make use of such a scalar was subsequently presented in [6]. Although the problems associated with the scalar are gone, the other objections in [22] probably remain. Nevertheless, since this action exhibits some new features it might be of some interest. The action is ($\mathscr{F}$ is an independent six-form field strength)

$$S_{T5} = \int d^6\xi\sqrt{-g}\lambda(1 + \frac{1}{12}F_{ijk}F^{ijk} - \frac{1}{24}k_{ij}k^{ij} + \frac{1}{72}(trk)^2 - (*\mathscr{F})^2) \tag{3.4}$$

where $k_{ij} = \frac{1}{2}F_i{}^{jk}F_{jkl}$. Note that, as explained in [6], the self-duality relation

$$-(*\mathscr{F}) * F_{ijk} = F_{ijk} - \frac{1}{2}k_{[i}{}^{l}F_{jk]l} + \frac{1}{6}(trk)F_{ijk} \tag{3.5}$$

does not arise as a field equation but as a result of demanding κ-symmetry, and must not be inserted into the action.

4. Superworld sheets in target superspace

It is possible to reformulate the brane dynamics, following from actions of the kinds described in the previous section, in terms of world sheet superfields. This can be done by embedding superworld sheets into target superspace. Besides making the world sheet supersymmetry manifest this has the further advantages of explaining the origin of κ-symmetry and the projection matrix [23] as well as of providing the connection between superembeddings on one hand, and Goldstone fermions and non-linearly realized supersymmetries on the supersheets on the other hand [24, 10].

In [24] Bagger and Galperin showed how the 4d Born-Infeld action for an abelian gauge field can be obtained by embedding a (4d,N=1) superspace into a (4d,N=2) one. The broken fermionic translations turn into non-linear supersymmetry transformations on the Goldstone fermions arising from half of the fermionic coordinates that are turned into dependent variables. The non-linearities of the Born-Infeld action are then seen to be a consequence of demanding consistency with the extra non-linear supersymmetries. In [10] this programme was taken over to the T5-brane in 7d superspace. This led to an equation for one of the supertorsion components whose rather complicated solution indicates that this way of analyzing this system is not the most efficient one. Fortunately once it is realized that this form of the torsion equation can also be obtained in the much more general formalism known as the embedding formalism developed in [8, 9] these problems can be circumvented.

The central equation is the torsion pullback equation (8,9,10)

$$\mathscr{D}_A \mathscr{E}_B{}^{\underline{C}} - (-1)^{AB} \mathscr{D}_B \mathscr{E}_A{}^{\underline{C}} + T_{AB}{}^C \mathscr{E}_C{}^{\underline{C}} = (-1)^{A(A+\underline{B})} \mathscr{E}_B{}^{\underline{B}} \mathscr{E}_A{}^{\underline{A}} T_{\underline{AB}}{}^{\underline{C}} \tag{4.1}$$

where underlined indices refer to target superspace while the other indices are connected with the superworld sheet. As shown in [23, 10] inserting constraints on the torsion components turns this equation into the equations of motion for the world sheet fields. In particular, the highly non-linear dynamics of the T5-branes in 11d [23] and in 7d [10] can be obtained this way.

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